KS5 Curriculum Map – Further Mathematics:

Topic	Substantive Knowledge This is the specific, factual content for the topic, which should be connected into a careful sequence of learning.	Disciplinary Knowledge (Skills) This is the action taken within a particular topic in order to gain substantive knowledge.	Assessment Opportunities What assessments will be used to measure student progress?
Proof by Induction	• Prove results about sums of series • Prove results about divisibility • Prove results about matrices	• Understand the principle of proof by mathematical induction • Construct proofs using mathematical induction in the context of sums of series, divisibility, and matrices.	• Controlled Homework • Baseline 5 assessment (November) • Baseline 6 Mock exams (January) • Baseline 7 Full FM Mocks (Post-Easter)
Complex Numbers	• Definition of complex numbers and complex conjugates • Perform basic operations with complex numbers • Solve polynomial equations with complex roots • Modulus-argument form • Argand diagrams, including loci and regions • Exponential form • De Moivre's theorem • Find nth roots of a complex number • Solve geometric problems	• Understand and use the 'real part' and 'imaginary part' of a complex number, and the complex conjugate, • Represent complex numbers on an argand diagram • Add, subtract, multiply and divide complex numbers in the form $x + iy$ with x and y real. • Solve quadratic, cubic and quartic equations with real coefficients and complex roots • Understand the meaning of, and find, the modulus and argument of complex numbers; know what this represents on an argand diagram • Convert between the Cartesian form and the modulus-argument form of a complex number; multiply and divide complex numbers in modulus-argument form.	• Controlled Homework • Baseline 5 assessment (November) • Baseline 6 Mock exams (January) • Baseline 7 Full FM Mocks (Post-Easter)

		• Construct and interpret simple loci in the argand diagram. • Know and use the definition of complex numbers in exponential form, and convert between cartesian/modulus-argument form and exponential form. • Understand De Moivre's theorem and use it to find multiple angle formulae and sums of series • Find the n distinct nth roots of $re^{i\theta}$ for $r \neq 0$ and know that they form the vertices of a regular • n-gon in the Argand diagram. • Use complex roots of unity to solve geometric problems	
Matrices	• Understand the concept of a matrix and perform matrix calculations • Find the inverse of a matrix and use this to solve problems. • Represent linear transformations as matrices. • Perform transformations in 2D and 3D using matrices.	• Understand the concept of a matrix, and define the zero and identity matrices • Add, subtract and multiply matrices • Multiply a matrix by a scalar • Calculate the determinant of 2x2 and 3x3 matrices • Understand and uses singular and non-singular matrices, and use this to find the inverse of 2x2 and 3x3 matrices • Use matrices to solve systems of equations • Interpret geometrically the solution and failure of solution of three simultaneous linear equations. • Understand the properties of linear transformations and represent them using matrices • Perform single and successive transformations in two dimensions using matrices, including reflections in coordinate axes, rotation through any angle about the origin, stretches parallel to the x and y axes and enlargements about centre (0,0).	• Controlled Homework • Baseline 5 assessment (November) • Baseline 6 Mock exams (January) • Baseline 7 Full FM Mocks (Post-Easter)

		• Perform single transformations in three dimensions confined to reflections in the coordinate axes and rotations about the coordinate axes. • Find invariant points and invariant lines for transformations. • Interpret the determinant as the area (or volume) scale factor in transformations.	
Vectors	• Equation of a line in three dimensions • Equation of a plane in three dimensions • Scalar product • Angles between lines and planes • Points of intersection • Finding perpendiculars	• Understand and use the vector and cartesian forms of an equation of a straight line in 3D • Understand and use the vector and cartesian forms of an equation of a plane • Calculate the scalar product and use it to express the equation of a plane, and to calculate the angle between two lines, the angle between two planes, and the angle between a line and a plane • Check whether vectors are perpendicular by using the scalar product. • Find the intersection of a line and a plane • Calculate the perpendicular distance between two lines, from a point to a line, and from a point to a plane.	• Controlled Homework • Baseline 5 assessment (November) • Baseline 6 Mock exams (January) • Baseline 7 Full FM Mocks (Post-Easter)
Momentum and Impulse	• Momentum in one direction • Conservation of momentum • Momentum as a vector	• Understand the meaning of 'momentum' and 'impulse', and calculate the momentum of a particle and the impulse of a force • Understand and use the impulse-momentum principle. • Understand the principle of conservation of momentum, as applied to two spheres, and use it to solve problems involving collisions. • Recognise that momentum can be represented as a vector. • Apply the impulse-momentum principle in vector form.	• Controlled Homework • Baseline 5 assessment (November) • Baseline 6 Mock exams (January) • Baseline 7 Full FM Mocks (Post-Easter)

Work, Energy, Power	• Work done • Kinetic and potential energy • Conservation of mechanical energy and the work-energy principle • Power	• Calculate the work done by a force when its point of application moves • Calculate the kinetic energy of a moving particle and the potential energy of a particle • Use the principle of conservation of mechanical energy and the work-energy principle • Solve problems involving motion under variable resistance and/or motion along an inclined plane. • Calculate the power developed by an engine.	• Controlled Homework • Baseline 6 Mock exams (January) • Baseline 7 Full FM Mocks (Post-Easter)
Elastic Strings and Springs and Elastic Energy	• Elastic strings and springs and Hooke's Law • Energy stored in an elastic string or spring • Problems involving elastic energy	• Use Hooke's Law to solve equilibrium problems involving elastic strings or springs • Use Hooke's Law to solve dynamics problems involving elastic strings or springs • Find the energy stored in an elastic string or spring. • Solve problems involving kinetic energy, potential energy and elastic energy using the principle of mechanical energy and the work-energy principle	• Controlled Homework • Baseline 6 Mock exams (January) • Baseline 7 Full FM Mocks (Post-Easter)
Elastic collisions in one dimension	• Direct impact and Newton's law of restitution • Direct collision with a smooth plane • Loss of kinetic energy • Successive direct impacts	• Understand and use Newton's Law of restitution and know that $\boldsymbol{e}$ is the coefficient of restitution and takes values between 0 and 1 inclusive. • Solve problems involving the direct impact of two particles by using the principle of conservation of momentum and Newton's law of restitution • Apply Newton's law of restitution to problems involving the direct collision of a particle with a smooth plane surface • Find the change in energy due to an impact or the application of an impulse. • Solve problems involving successive direct impacts.	• Controlled Homework • Baseline 6 Mock exams (January) • Baseline 7 Full FM Mocks (Post-Easter)

Elastic collisions in two dimensions	• Oblique impacts with a fixed surface • Successive oblique impacts • Loss of kinetic energy • Oblique impact of smooth spheres	• Apply knowledge of momentum, impulse and restitution to smooth collisions in two dimensions. This may include problems set in vector form. • Solve problems involving the oblique impact of a smooth sphere with a fixed surface. • Solve problems involving successive oblique impacts of a sphere with smooth plane surfaces. • Solve problems involving the oblique impact of two smooth spheres • Find the change in energy due to an impact or the application of an impulse in two dimensions.	• Controlled Homework • Baseline 6 Mock exams (January) • Baseline 7 Full FM Mocks (Post-Easter)
Calculus	• Volumes of revolution • Improper integrals • The mean value of a function • Integration using partial fractions • Differentiation of inverse trigonometric functions • Integration with inverse trigonometric functions	• Derive formulae for, and calculate, volumes of revolution. This includes revolutions around both the x axis and the y axis, and for curves given in cartesian and parametric form. • Model real-life applications of volumes of revolution. • Evaluate improper integrals where either the integrand is undefined at a value in the range of integration or range of integration extends to infinity. • Understand and evaluate the mean value of a function. • Integrate using partial fractions, extending to include quadratic factors $ax^2 + c$ in the denominator. • Differentiate inverse trigonometric functions • Integrate functions of the form $(a^2 - x^2)^{-\frac{1}{2}}$ and $(a^2 - x^2)^{-1}$ and be able to choose trigonometric substitutions to integrate associated functions.	• Controlled Homework • Baseline 6 Mock exams (January) • Baseline 7 Full FM Mocks (Post-Easter)

Hyperbolic functions	• Introduction to hyperbolic functions • Inverse hyperbolic functions • Identities and equations • Calculus of hyperbolic functions	• Understand the definition of hyperbolic functions $\sinh x$, $\cosh x$ and $\tanh x$, including their domains and ranges, and be able to sketch their graphs. • Understand and be able to use the definitions of the inverse hyperbolic functions and their domains and ranges. • Derive and use the logarithmic forms of the inverse hyperbolic functions. • Integrate functions of the form • $(x^2 - a^2)^{-\frac{1}{2}}$ and $(x^2 + a^2)^{-\frac{1}{2}}$ and be able to choose suitable hyperbolic substitutions to integrate associated functions. • Differentiate and integrate hyperbolic functions	• Controlled Homework • Baseline 6 Mock exams (January) • Baseline 7 Full FM Mocks (Post-Easter)
Differential equations	• First-order differential equations • Second-order homogeneous differential equations • Second-order non-homogeneous differential equations • Boundary conditions with second order differential equations • Modelling with first-order differential equations • Simple harmonic motion • Damped and forced harmonic motion • Coupled first-order simultaneous differential equations	• Find and use an integrating factor to solve some first order differential equations and recognise when it is appropriate to do so. • Use boundary conditions to find particular solutions to differential equations. • Use differential equations in modelling in kinematics and in other contexts. • Solve second-order homogeneous equations by using the auxiliary equation. • Solve second-order non-homogeneous differential equations by solving the homogeneous case and adding a particular integral to the complementary function (in cases where the right-hand side of the equation is a polynomial, exponential or trigonometric function) • Understand and use the relationship between the cases when the discriminant of the auxiliary equation is positive, zero and negative and the form of solution of the differential equation	• Controlled Homework • Controlled Question 2 (March) • Baseline 7 Full FM Mocks (Post-Easter)

		• Understand the concepts behind simple harmonic motion, solve the equation for simple harmonic motion and relate the solution to the motion. • Model damped oscillations using second-order differential equations and interpret their solutions. • Analyse and interpret models of situations with one independent variable and two dependent variables as a pair of coupled first-order simultaneous equations and be able to solve them, for example predator-prey models.	
Discrete Probability distributions	• Expected value and variance of a discrete random variable • Expected value and variance of a function of X. • Solving problems involving random variables.	• Calculation of the mean and variance of discrete probability distributions. • Extension of expected value function to include E(g(X)). • Use these calculations to assess the suitability of models.	• Controlled Homework • Controlled Question 2 (March) • Baseline 7 Full FM Mocks (Post-Easter)
Poisson and Binomial distributions	• Introduction to the Poisson distribution • Modelling with the Poisson distribution • Adding Poisson distributions • Mean and variance of Binomial and Poisson distributions • Using the Poisson distribution to approximate the Binomial distribution	• Understand and use the Poisson distribution to solve problems. • Use the Poisson distribution to model a real-world situation and comment critically on the appropriateness. • Use the additive property of the Poisson distributions. • Calculate the mean and variance of the Binomial and Poisson distributions. • Know the conditions under which the Binomial distribution can be approximated by the Poisson distribution and use this approximation when it is appropriate to do so.	• Controlled Homework • Controlled Question 2 (March) • Baseline 7 Full FM Mocks (Post-Easter)

Geometric and negative binomial distributions	• Introduction to the geometric distribution, including mean and variance • Introduction to the negative binomial distribution, including mean and variance of a negative binomial distribution	• Understand the models leading to the geometric distribution and the negative binomial distribution. • Calculate the mean and variance of the geometric distribution and the negative binomial distribution.	• Controlled Homework • Baseline 7 Full FM Mocks (Post-Easter)
Hypothesis testing	• Understand and conduct hypothesis tests for the Poisson distribution and geometric distributions	• Extend ideas of hypothesis testing to test for the mean of a Poisson distribution • Find critical regions for a Poisson distribution • Extend ideas of hypothesis testing to test for the parameter p of a geometric distribution • Find critical regions for a geometric distribution	• Controlled Homework • Baseline 7 Full FM Mocks (Post-Easter)
Central Limit Theorem	• Introduction to the Central Limit Theorem • Application of the Central Limit Theorem to other distributions	• Understand the concepts underpinning the Central Limit Theorem • Apply the Central Limit Theorem to the Binomial, Normal, Poisson, geometric and negative binomial distributions.	• Controlled Homework • Baseline 7 Full FM Mocks (Post-Easter)
Chi Squared Tests	• Goodness of fit • Degrees of freedom and the chi-squared family of distributions • Testing a hypothesis • Testing the goodness of fit with discrete data • Using contingency tables • Apply goodness-of-fit tests to geometric distributions.	• Form hypotheses about how well a distribution fits as a model for an observed frequency distribution and measure goodness of fit of a model to observed data • Understand degrees of freedom and use the chi-squared family of distributions • Be able to test a hypothesis • Apply goodness-of-fit tests to discrete data • Use contingency tables • Apply goodness-of-fit tests to uniform, binomial Poisson and geometric distributions. • Find p-values from calculators and use tables of values to find critical values.	• Controlled Homework • Baseline 7 Full FM Mocks (Post-Easter)

Probability Generating Functions	• Introduction to probability generating functions • Probability generating functions of standard distributions • Mean and variance of a distribution • Sums of independent random variables	• Understand the definitions, derivations and applications of probability generating functions. • Use the probability generating function for the negative binomial, geometric, binomial and Poisson distributions. • Use probability generating functions to find the mean and variance and know the proofs of standard results. • Find probability generating functions of the sum of independent random variables.	• Controlled Homework • Baseline 7 Full FM Mocks (Post-Easter)
Quality of Tests	• Type I and Type II errors • Finding Type I and Type II errors • Calculate the size and power of a test • The power function	• Know the definition of Type I and Type II errors • Calculate the probability of a Type I or Type II error using the normal distribution • Find the size and power of a test and draw a graph of the power function for a test. • Use of Type I and Type II errors and power function to indicate the effectiveness of statistical tests (including binomial, normal, Poisson, geometric and negative binomial).	• Controlled Homework • Baseline 7 Full FM Mocks (Post-Easter)
Further algebra and functions	• Roots of polynomials • Sums of series • Method of differences • Maclaurin series and series expansions of compound functions	• Understand and use the relationship between roots and coefficients of polynomial equations, up to and including quartic equations. • Form a polynomial equation whose roots are a linear transformation of the roots of a given polynomial (of at least cubic degree). • Understand and use formulae for the sums of integers, square and cubes and use these to sum other series. • Understand and use the method of differences for summation of series including use of partial fractions • Find the Maclaurin series of a function including the general term. • Recognise and use the Maclaurin series for e^x, $\ln(1+x)$, $\sin x$, $\cos x$ and $(1+x)^n$,	• Controlled Homework • Baseline 7 Full FM Mocks (Post-Easter)

		and be aware of the range of values of x for which they are required.	
Polar coordinates	• Polar coordinates and equations • Sketching curves • Area enclosed by a polar curve • Tangents to polar curves	• Understand and use polar coordinates and be able to convert between polar and cartesian coordinates • Sketch curves with r given as a function of θ, including use of trigonometric functions. • Find the area enclosed by a polar curve • Find tangents parallel to, or perpendicular to, the initial line.	• Controlled Homework • Baseline 7 Full FM Mocks (Post-Easter)